



# **PiXL Gateway: Progression - Mathematics**

## **Solutions**

### Basic Skills check

### ANSWERS

1. Expand the brackets  $(2x - 4)(-4 + x)$   $2x^2 - 12x + 16$

2. Given  $f(x) = x^2 + 5x - 2$  find the value of  $f(4)$   $34$

3. Solve the simultaneous equations.

$$3x - 4y = 20$$

$$5x + 5y = 10$$

$$x = 4$$

$$y = -2$$

4. Solve each of these equations.

(i)  $4x - 3 = 15$   $x = 4.5$

(ii)  $\frac{y}{3} + 4 = 9$   $y = 15$

(iii)  $5m - 8 = 2m + 13$   $m = 7$

5. Simplify  $(3 + \sqrt{2})(3 - \sqrt{2})$   $7$

6. Express  $\frac{1+\sqrt{2}}{3-\sqrt{2}}$  in the form  $a + b\sqrt{2}$  where  $a$  and  $b$  are rational.

$$\frac{5}{7} + \frac{4}{7}\sqrt{2}$$

7. Simplify  $\frac{(2x^2y^3z)^5}{4y^2z}$   $8x^{10}y^{13}z^4$

8. A (0 , 2), B (7 , 9) and C (6 , 10) are three points.

(i) Show that AB and BC are perpendicular.

$$\text{Grad AB} = 1$$

$$\text{Grad BC} = -1$$

product of gradients = -1 hence perpendicular

- (ii) Find the length of AC. Length 10
9. Sketch the graph of  $y = 9 - x^2$   
 Negative quadratic curve  
 Intercept (0 , 9)  
 Through (3 , 0) and (-3 , 0)
10. The curve  $y = x^2 - 4$  is translated by  $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$   
 Write down an equation for the translated curve. You need not simplify your answer.  
 $y = (x - 2)^2 - 4$

11. Given that  $\cos\theta = \frac{1}{3}$  and  $\theta$  is acute, find the exact value of  $\tan\theta$ .  
 $\sqrt{8}$  or  $2\sqrt{2}$

12. Solve

(i) $x^2 - 36 \leq 0$	$-6 \leq x \leq 6$
(ii) $9x^2 - 25 \geq 0$	$x \leq -\frac{5}{3}, x \geq \frac{5}{3}$
(iii) $3x^2 + 10x < 0$	$-\frac{10}{3} < x < 0$

13. Prove that the square of an odd number is also odd.

$$\begin{aligned}
 &2n + 1 \text{ is an odd number} \\
 &(2n + 1)^2 = 4n^2 + 4n + 1 \\
 &4n^2 + 4n = 4(n^2 + n) \text{ is even} \\
 &\text{So } 4n^2 + 4n + 1 \text{ odd}
 \end{aligned}$$

14. Caleb either walks to school or travels by bus.

The probability that he walks to school is 0.75

If he walks to school, the probability that he will be late is 0.3

If he travels to school by bus, the probability that he will be late is 0.1

Work out the probability that he will not be late.

$$0.75 \times 0.7 = 0.525 \text{ or } 0.25 \times 0.9 = 0.225$$

$$0.525 + 0.225$$

$$= 0.75$$

## Problem Solving

1. Two numbers have a product of 44 and a mean of 7.5

Use an algebraic method to find the numbers.

You must show all your working.

1st number:  $x$  then 2nd number:  $\frac{44}{x}$

$$\frac{x + \frac{44}{x}}{2} = 7.5$$

$$x + \frac{44}{x} = 15$$

$$x^2 + 44 = 15x$$

$$x^2 - 15x + 44 = 0$$

$$(x - 11)(x - 4) = 0$$

$$x = 11 \text{ or } 4 \quad \text{so the numbers are 11 and 4}$$

2. In a parallel circuit, the total resistance is given by the formula  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Make  $R_1$  the subject of the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_1 \times R_2 = R \times R_2 + R \times R_1$$

$$R_1 \times R_2 - R \times R_1 = R \times R_2$$

$$R_1(R_2 - R) = R \times R_2$$

$$R_1 = \frac{R \times R_2}{(R_2 - R)}$$

3. Sarah intended to spend £6.00 on prizes for her class but each prize cost her 10p more than expected, so she had to buy 5 fewer prizes.  
Calculate the cost of each prize.

*Let  $x$  be no. of original prizes &  $y$  be the price of each original prize*

$$xy = 600 \quad \Rightarrow \quad x = \frac{600}{y}$$

$$(x - 5)(y + 10) = 600$$

$$xy - 5y + 10x - 50 = 600$$

$$\frac{600y}{y} - 5y + \frac{6000}{y} - 50 = 600$$

$$5y + 50 - \frac{6000}{y} = 0$$

$$5y^2 + 50y - 6000 = 0$$

$$y^2 + 10y - 1200 = 0$$

$$(y + 40)(y - 30) = 0$$

*$y$  is a price so  $y = 30$*

*Cost of each prize =  $y + 10 = 40$ p*

4. Arthur and Florence are going to the theatre.  
Arthur buys 6 adult tickets and 2 child tickets and pays £39  
Florence buys 5 adult tickets and 3 child tickets and pays £36.50  
Work out the costs of both adult and child tickets.

$$6A + 2C = 39 \quad \times 3 \quad 18A + 6C = 117$$

$$5A + 3C = 36.50 \quad \times 2 \quad 10A + 6C = 73$$

*Subtracting equations:  $8A = 44$*

$$A = 5.5$$

*Substitute:  $(6 \times 5.5) + 2C = 39$*

$$C = 3$$

*Adult ticket = £5.50      Child ticket = £3*

5. Colin has made a mistake in his 'simplifying surds' homework. Explain his error and give the correct answer.

$$4\sqrt{3} \times 5\sqrt{12} = 20\sqrt{36}$$

Valid explanation- correct answer 120

6. Below is a sketch of  $f(x)$ .

The coordinates of  $P$  are  $(0, -2)$

State the coordinates of  $P$  after each translation:

(i)  $g(x) = f(x) + 1$

$P' = (0, -1)$

(ii)  $h(x) = f(x - 2)$

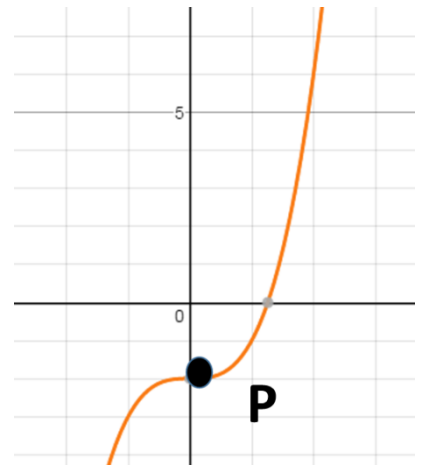
$P' = (2, -2)$

(iii)  $j(x) = -f(x)$

$P' = (0, 2)$

(iv)  $k(x) = f(-x)$

$P' = (0, -2)$



7. The equation of a curve is  $y = f(x)$  where  $f(x) = x^2 - 4x + 5$   
 C is the minimum point of the curve.  
 (i) Find the coordinates of C after the transformation  $f(x + 1) + 2$

$$f(x) = (x - 2)^2 + 1$$

Before transformation C is (2, 1)

After transformation C is (1, 3)

- (ii) Determine if  $f(x - 3) - 1 = 0$  has any real roots.  
 Give reasons for your answer.

Min point for  $f(x - 3) - 1$  at (5, 0)

Hence it has a single, repeated root at  $x = 5$

8. A piece of land is the shape of an isosceles triangle with sides 7.5m, 7.5m and 11m.  
 Turf can be bought for £11.99 per 5m<sup>2</sup> roll.  
 How much will it cost to turf the piece of land?

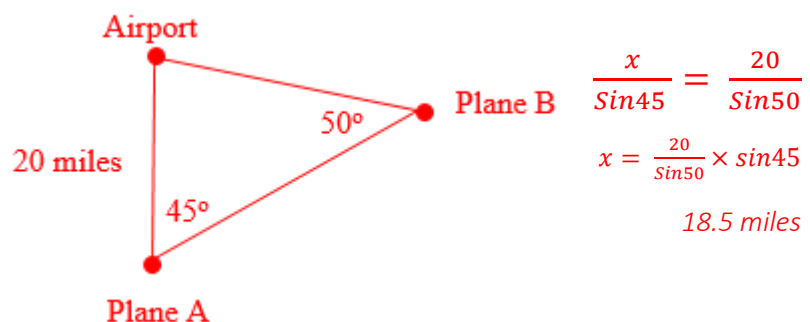
$$\sqrt{(7.5^2 - 5.5^2)} = \sqrt{26}m \text{ Length of the land}$$

$$\text{Area of land} = 11 \times \sqrt{26} \div 2 = 28.04m^2$$

Each roll covers 5m<sup>2</sup> so need 6 rolls

Need to buy 6 rolls at £11.99 per roll Total cost = £71.94

9. Plane A is flying directly toward the airport which is 20 miles away. The pilot notices a second plane, B, 45° to her right. Plane B is also flying directly towards the airport. The pilot of plane B calculates that plane A is 50° to his left. Based on that information how far is plane B from the airport? Give your answer to 3 significant figures.



10. A farmer has a triangular field. He knows one side measures 450m and another 320m. The angle between these two sides measures  $80^\circ$ . The farmer wishes to use a fertiliser that costs £3.95 per container which covers  $1500\text{m}^2$ . How much will it cost to use the fertiliser on this field?

$$\text{Area of field} = 0.5 \times 450 \times 320 \times \sin 80$$

$$\text{Area of field} = 70906\text{m}^2 \text{ (to the nearest sq.m)}$$

$$70906 \div 1500 = 47.27$$

Needs to buy 48 containers

$$£3.95 \times 48 = £189.60$$

11. Katie chooses a two-digit number, where the digits are different, reverses the digits, and subtracts the smaller number from the larger.

For example

$$42 - 24 = 18$$

She tries several different numbers and finds the answer is never a prime number.

Prove that Katie can never get an answer that is a prime number.

My numbers are

$$10a + b \text{ and } 10b + a$$

$$10a + b - (10b + a)$$

$$= 10a - 10b + b - a$$

$$= 9a - 9b$$

$$= 9(a - b)$$

- Attempts to write an expression for the first number
- Writes the correct expression for the first number
- Writes the correct expression for the second number
- Attempts to subtract the expressions
- Simplifies the result
- Factorises
- Makes the statement

The answer is always a multiple of 9

12. The Venn diagram shows the ice-cream flavours chosen by a group of 44 children at a party.

The choices are strawberry (S), choc-chip (C) and toffee (T). A child is picked at random.



Work out :

(i)  $P(S)$

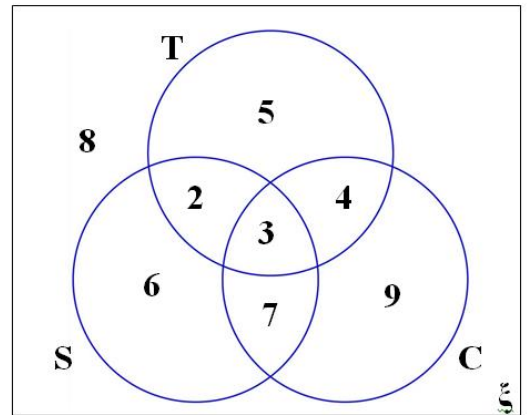
$$= \frac{9}{22}$$

(ii)  $P(T \cap C | C)$

$$= \frac{7}{23}$$

(iii)  $P(C | S \cup T)$

$$= \frac{14}{27}$$





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